

Closing next wed: HW\_2A,2B,2C  
Office Hours: 1:30-3:00pm in Com.B-006

Quick review:

**Def'n:** The "signed" area between  $f(x)$  and the  $x$ -axis from  $x = a$  to  $x = b$  is the *definite integral*:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x,$$

where  $\Delta x = \frac{b-a}{n}$  and  $x_i = a + i\Delta x$

**FTOC(1):** Areas are antiderivatives!

$$\frac{d}{dx} \left( \int_a^x f(t) dt \right) = f(x)$$

**FTOC(2):** If  $F(x)$  is any antideriv. of  $f(x)$ ,

$$\int_a^b f(x) dx = F(b) - F(a)$$

**Entry Task:** Evaluate

$$\begin{aligned} & \int_0^4 e^x + \sqrt{x^3} dx \quad \leftarrow x^{3/2} \\ &= e^x + \frac{2}{5} x^{5/2} \Big|_0^4 \\ &= \left[ e^4 + \frac{2}{5} (4)^{5/2} \right] - \left[ e^0 + \frac{2}{5} (0)^{5/2} \right] \\ &= e^4 + \frac{64}{5} - 1 = \boxed{e^4 + \frac{59}{5}} \end{aligned}$$

$$\begin{aligned} & \int_3^6 \frac{4}{x} - \frac{2}{x^2} dx = \int_3^6 4 \frac{1}{x} - 2 x^{-2} dx \\ &= 4 \ln|x| - 2 \frac{1}{x^{-1}} \Big|_3^6 \\ &= 4 \ln(x) + \frac{2}{x} \Big|_3^6 \\ &= \left[ 4 \ln(6) + \frac{2}{6} \right] - \left[ 4 \ln(3) + \frac{2}{3} \right] \\ &= 4 \ln(6) + \frac{1}{3} - 4 \ln(3) - \frac{2}{3} \\ &= 4 \ln(6) - 4 \ln(3) - \frac{1}{3} \\ &= \boxed{4 \ln(2) - \frac{1}{3}} \end{aligned}$$

## 5.4 The Indefinite Integral and Net/Total Change

**Def'n:** The **indefinite integral** of  $f(x)$  is defined to be the general antiderivative of  $f(x)$ . And we write

$$\int f(x)dx = F(x) + C,$$

where  $F(x)$  is any antiderivative of  $f(x)$ .

Ex)

$$\int x^3 dx = \frac{1}{4} x^4 + C = \text{a function}$$

$$\int_0^1 x^3 dx = \left. \frac{1}{4} x^4 \right|_0^1 = \frac{1}{4} = \text{a number}$$

A brief pause to discuss current integration methods. We can currently find antiderivatives for sums and constant multiples of functions **directly** from our integration table.

Examples (we can currently do):

$$1. \int 6e^x + 4x - 5\sqrt{x} \, dx$$

$$= 6(e^x) + 4\left(\frac{1}{2}x^2\right) - 5\left(\frac{2}{3}x^{3/2}\right) + C$$

$$= 6e^x + 2x^2 - \frac{10}{3}x^{3/2} + C$$

$$2. \int 6\sec^2(x) - \frac{9}{x^4} \, dx$$

$$\int 6\sec^2(x) - 9x^{-4} \, dx$$

$$= 6(\tan(x)) - 9\left(\frac{1}{-3}x^{-3}\right) + C$$

$$= 6\tan(x) + \frac{3}{x^3} + C$$

## 4.9: LIST OF GENERAL ANTIDERIVATIVES

$$\int ||^x dx = \frac{1}{\ln(||)} ||^x + C$$

**FUNCTION**                      **ANTIDERIVATIVE**

$$f(x) = x^n \quad (n \neq -1) \qquad F(x) = \frac{1}{n+1} x^{n+1} + C$$

$$f(x) = x^{-1} = \frac{1}{x} \qquad F(x) = \ln|x| + C$$

$$f(x) = e^x \qquad e = 2.7182818 \dots \qquad F(x) = e^x + C$$

$$f(x) = a^x \qquad F(x) = \frac{1}{\ln(a)} a^x + C$$

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

$$f(x) = \cos(x) \qquad F(x) = \sin(x) + C$$

$$f(x) = \sec^2(x) \qquad F(x) = \tan(x) + C$$

$$f(x) = \frac{1}{\cos(x)} \frac{\sin(x)}{\cos(x)} = \sec(x) \tan(x) \qquad F(x) = \sec(x) + C$$

$$f(x) = \sin(x) \qquad F(x) = -\cos(x) + C$$

$$f(x) = \csc^2(x) \qquad F(x) = -\cot(x) + C$$

$$f(x) = \frac{1}{\sin(x)} \frac{\cos(x)}{\sin(x)} = \csc(x) \cot(x) \qquad F(x) = -\csc(x) + C$$

$$f(x) = \frac{1}{1+x^2} \qquad F(x) = \tan^{-1}(x) + C$$

$$f(x) = a^x \qquad F(x) = \frac{1}{\ln(a)} a^x + C$$

Examples we **cannot** currently do (but will be able to do later in the term):

$$\int x e^{3x} dx; \quad \int \tan(x) dx$$
$$\int x \sin(x^2) dx; \quad \int \frac{x^2 - x + 6}{x^3 + 3x} dx$$
$$\int \frac{3}{x - 2\sqrt{x}} dx; \quad \int \frac{\sqrt{x^2 - 1}}{x^2} dx$$

Examples we will "never" be able to do:

$$\int e^{x^2} dx; \quad \int \sec(x^2) dx$$

$$\boxed{1} \int x^{-1/2} - 3 dx$$
$$= \boxed{2x^{1/2} - 3x + C}$$

Here are two that look bad but we can currently do them, why?

$$1. \int \frac{\sqrt{x} - 3x}{x} dx = \int \frac{1}{x} (\sqrt{x} - 3x) dx$$

$$2. \int \frac{\cos(x)}{1 - \cos^2(x)} dx$$

$$\boxed{2} \int \frac{\cos(x)}{\sin^2(x)} dx$$
$$= \int \frac{\cos(x)}{\sin(x)} \frac{1}{\sin(x)} dx$$
$$= \int \cot(x) \csc(x) dx$$
$$= \boxed{-\csc(x) + C}$$

What is the value of:

$$\int_{\pi/4}^{\pi/2} \frac{\cos(x)}{1 - \cos^2(x)} dx$$

$$= -\csc(x) \Big|_{\pi/4}^{\pi/2}$$

$$= [-\csc(\pi/2)] - [-\csc(\pi/4)]$$

$$= \left[ -\frac{1}{\sin(\pi/2)} \right] - \left[ -\frac{1}{\sin(\pi/4)} \right]$$

$$= -1 + \frac{1}{\sqrt{2}/2}$$

$$= -1 + \frac{2}{\sqrt{2}}$$

$$= -1 + \sqrt{2}$$

## Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

Let

$s(t)$  = 'location at time  $t$ '

$v(t)$  = 'velocity at time  $t$ '

pos.  $v(t)$  means moving up/right

neg.  $v(t)$  means moving down/left

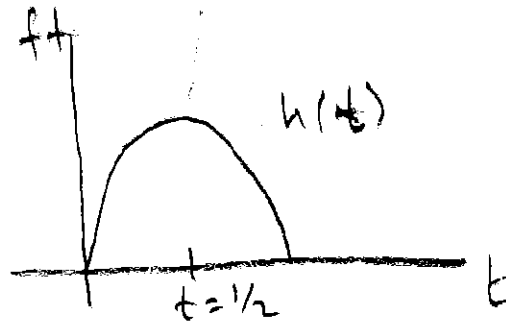
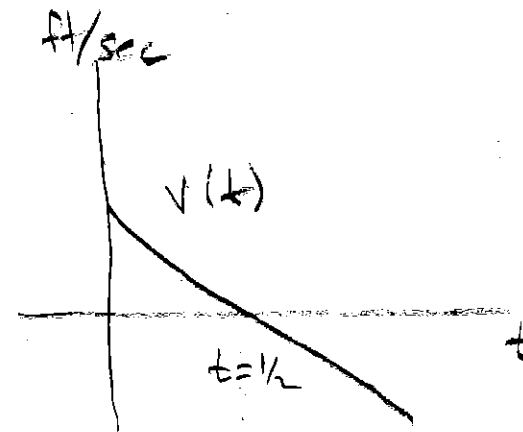
The FTC (part 2) says

$$\int_a^b v(t) dt = s(b) - s(a)$$

i.e.

'integral of velocity' = 'net change in dist'

We also call this the *displacement*.

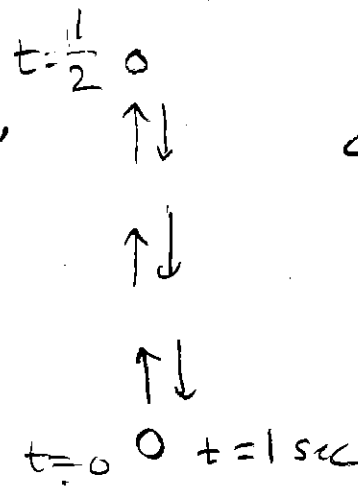


$$a(t) = -32$$

$$v(t) = -32t + 16$$

$$h(t) = -16t^2 + 16t + 0$$

ACTUAL VIEW OF OBJECT



$$\int_0^1 v(t) dt = 0$$

↑  
displacement

Thus, in general, the FTC(2) says the **net change** in  $f(x)$  from  $x = a$  to  $x = b$  is the integral of its **rate**.

That is:

$$\int_a^b \underbrace{f'(t)}_{\substack{\uparrow \\ \text{gal} \\ \text{hr}}} dt = \underbrace{f(b) - f(a)}_{\text{Change in gallons}}$$

$$\frac{\text{people}}{\text{yr}} \text{ yr} \quad \text{change in people}$$

etc.

⋮

⋮



We define **total change** in dist. by

$$\int_a^b |v(t)| dt$$

which we compute by

1. Solving  $v(t) = 0$  for  $t$ .
2. Splitting up the integral at these  $t$  values, dropping the absolute value and integrating separately.
3. Adding together as positive numbers.

Example:  $v(t) = t^2 - 2t - 8$  ft/sec  
Compute the total distance traveled from  $t = 1$  to  $t = 6$ .

$$\int_1^6 |t^2 - 2t - 8| dt$$

$$\begin{aligned} \square 1 \quad t^2 - 2t - 8 &\stackrel{?}{=} 0 \\ (t-4)(t+2) &= 0 \\ t &= 4 \quad \text{and} \quad t = -2 \end{aligned}$$

$$\square 2 \quad \int_1^4 t^2 - 2t - 8 dt = -18 \text{ ft}$$

$$\int_4^6 t^2 - 2t - 8 dt = \frac{44}{3} = 14.\bar{6} \text{ ft}$$

The object goes "left" 18 feet  
then "right" 14. $\bar{6}$  feet  
For

$$\boxed{\text{TOTAL DISTANCE} = 18 + 14.\bar{6} = 32.\bar{6} \text{ ft}}$$

NOTE:

$$\begin{aligned} \text{DISPLACEMENT} &= \int_1^6 t^2 - 2t - 8 dt = -18 + 14.\bar{6} \\ &= -3.\bar{3} \text{ ft} \end{aligned}$$